SUMMARY OF CEIOPS CALIBRATION WORK ON STANDARD FORMULA

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INTRODUCTION

Purpose

The purpose of this paper is to outline the methodology used to calibrate the "standard formula" in Solvency II. Given that Solvency II remains a work in progress, many of the details are not yet final. Nonetheless, the various documents that have been released by CEIOPS, coupled with decisions made by the European Commission in response to CEIOPS advice, provide some indication of the history and direction of calibration.

Background and History

CEIOPS is currently undertaking the fifth Quantitative Impact Study (QIS 5) regarding the standard formula and its calibration. QIS 1 (2005) and QIS 2 (2006) were aimed at understanding the level of prudence in statutory reserves (technical provisions) and the architecture of the standard formula, respectively. QIS 3 (2007) and QIS 4 (2009) undertook further evaluation of the calibration of the standard formula, and QIS 5 is expected to be the final QIS prior to implementation. At each step, CEIOPS has made revisions to the factors used in the standard formula. It is expected that further revisions will be made, prior to Solvency II implementation, based on information learned from QIS 5. Thus, this paper should be viewed as an attempt to outline the methodology that is currently being considered by CEIOPS, and not as a description of the final structure of the standard formula.

The Structure of Solvency II

Solvency II's standard formula will be used by those companies that do not use internal models to meet their regulatory capital requirements, i.e., to establish the Solvency Capital Requirement, or SCR. The SCR for an individual company is intended to be the approximately 99.5% Value at Risk (VaR) for that company, or a capital level that would result in a 1 in 200 probability of insolvency over a one-year time period. (See Appendix A for a description of VaR.)

Solvency II uses a building-block approach in its "standard formula" to compute capital requirements (or overall SCRs) for insurance companies. First, capital requirements are calculated for individual component risks. These amounts are then combined using correlation factors to obtain an overall capital requirement or SCR.

In some cases, the individual component risks are themselves composed of sub-risks, in which case, the capital requirements for the sub-risks are combined (using specified correlation factors) to determine the capital requirement for the relevant individual component risk.

To understand how Solvency II is calibrated, one must know the following: (1) the component risks and sub-risks that are used, (2) how these risks or sub-risks are individually calibrated, (3) how the correlation factors are determined, and (4) how the results are combined to produce the SCR.

1. The individual component risks include: market risk, counterparty default risk, life underwriting risk, health underwriting risk, non-life underwriting risk, and operational risk. Of these, market risk, life risk, non-life risk, and health risk contain two or more sub-risks.
2. The component risks and sub-risks have been individually calibrated by a variety of approaches, including empirical analysis and judgment. Many of the factors have been revised based on the QIS results. In the discussion that follows, two examples are provided – the calibration of equity risk (a sub-risk of the market risk module) and the calibration of mortality risk (a sub-risk in the life underwriting risk module).
3. The correlation factors were also chosen by a combination of empirical analysis (where possible) and judgment. (Note that CEIOPS elected not to use the customary Pearson product-moment measure. For a description of the general model used to determine correlation factors, see Appendix B.)
4. The next section on the SCR standard formula describes how the results are aggregated to produce the overall SCR.

This paper is not intended to provide a detailed explanation of the methodology used to determine the capital requirement and correlation factor for each individual risk and sub-risk. The Solvency II Calibration paper provides great detail on the analysis that was undertaken, and there is no attempt to restate it here. Rather, this paper is designed to provide a high-level overview of the methodology, along with selected examples, to illustrate the approach. This work includes two appendices which further elaborate on information provided in the paper.

SCR STANDARD FORMULA

As indicated earlier, the standard formula for Solvency II uses a building block approach. According to Paragraphs 3.1238 and 3.1239 of the Solvency II Calibration paper (CEIOPS-SEC-40-10, dated 15 April 2010),

> "[t]he standard formula … follows a modular approach. The overall risk which the insurance or reinsurance is exposed to is [partitioned] into [component] risks. For each[component] risk a capital requirement is determined."
>
> "The capital requirement for the overall risk is
>
> $$SCR_{overall} = \sqrt{\sum_{i.j} Corr_{i,j} \times SCR_i \times SCR_j} \quad (1)$$
>
> where i and j run over all [of the component] risks and $Corr_{i,j}$ denotes the entries of the correlation matrix, i.e., the correlation parameters."

So, in order to compute the overall solvency capital requirement, $SCR_{overall}$, one needs (1) the SCR for each component risk and (2) the correlation matrix. As will be seen, considerable judgment was used by CEIOPS to determine a SCR for each component risk and the correlation factors that are used to combine them.

The individual component risks considered in the overall formula are:

1. Market Risk,
2. Counterparty Default Risk,
3. Life Underwriting Risk,
4. Health Underwriting Risk,
5. Non-Life Underwriting Risk, and
6. Operational Risk.

The correlation factors developed by CEIOPS in the Solvency II Calibration paper for the first five component risks are presented in Table 1 below:

TABLE 1
OVERALL CORRELATION MATRIX*

	Market Risk	Counter-party Default Risk	Life Underwriting Risk	Health Underwriting Risk	Non-life Underwriting Risk
Market Risk	100%	25%	25%	25%	25%
Default		100%	25%	25%	50%
Life			100%	25%	0%
Health				100%	0%
Non-life					100%

Because there is no allowance for a diversification benefit between operational risk and any of the other risks, all of the correlation factors involving operational risk are considered to be 100% and are omitted from Table 1.

MARKET RISK COMPONENT

To illustrate the methodology used by CEIOPS, consider first the market risk component/module of the overall SCR. The market risk module involves six sub-risks:

- interest rate risk,
- equity risk,
- property risk,
- spread risk,
- currency risk, and
- concentration risk.

The capital requirement for the market component risk is obtained by aggregating the individual sub-risk SCR terms in conjunction with the correlation factors according to:

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j} \quad (2)$$

where i and j run over all the market component sub-risks and $Corr_{i,j}$ denotes the entries of the correlation matrix for the market component sub-risks. CEIOPS estimated each SCR each sub-risk in such a manner that it is "calibrated" at a 99.5% VaR.

The correlation factors for these sub-risks are presented in Table 2 below:

TABLE 2
CEIOPS MARKET RISK CORRELATION MATRIX*

	Interest Rate	Equity	Property	Spread	Currency	Concentration
Interest Rate	100%	50%/0%	50%/0%	50%/0%	50%	50%
Equity		100%	75%	75%	50%	50%
Property			100%	50%	50%	50%
Spread				100%	50%	50%
Currency					100%	50%
Concentration						100%

*CEIOPS is proposing a two-sided correlation between interest rate and each of three other risks. The choice of the correlation factor depends on whether a rise or fall in interest rates is the crucial factor. In the event the insurer would have adverse results if interest rates fell, then the 50% correlation factor should be used. In the event the insurer would have adverse results if interest rates rose, then the 0% correlation factor should be used.

In the next two sections, we illustrate the CEIOPS approaches to estimating (1) the SCR factors/stress levels and (2) the correlation factors.

Example of Calculation of SCR Factor (Stress Level) for Individual Sub-risk

On pages 37-41, CEIOPS describes the approach it took to estimate the standard equity capital charge for global equities. CEIOPS was concerned that financial distributions have "fat tails". Such distributions, by definition, have more weight in their right tails than do normal distributions with identical means and variances. CEIOPS considered a number of items including the following:

- The empirical 99.5% VaR for the Morgan Stanley Capital International (MSCI) World Developed (Market) Price Equity Index, using daily data beginning with the creation of the index in 1970 and ending in 2009, was -44.25%.

- The empirical 99.5% VaR for the MSCI World Equity Total Return Index using monthly data beginning in 1970 was -42.12%.

- Between 2008 and 2009, many "well-diversified equity portfolios (i.e., mimicking the MSCI [World Equity Index]) have halved in value".

This analysis led CEIOPS to conclude that "a stress of 45%" was reasonable "for global equities". (Interestingly, for purposes of QIS 5, the European Commission has reduced the stress factor to 39%.)

Example of Calculation of Correlation Factor[1]

To give the reader an idea of how these factors were computed, we briefly describe CEIOPS' efforts to compute the correlation factor between the equity and spread sub-risks of market risk. The CEIOPS researchers considered a wide variety of possible solutions. In the end the result chosen was based on 21 data points encompassing the period from September 9, 1998 to October 28, 1998. This was during and/or immediately after the financial crisis resulting from the demise of Long Term Capital Management. This was obviously a period of extreme stress. (In fact, CEIOPS describes this period as one of "very extreme stress".) The specific series used were (1) the Morgan Stanley Capital International (MSCI) world index for equities and (2) "the spread to gilts on UK AA rated corporate bonds". It turned out that the empirical correlation factor over this period was reported as exactly 75% and this was the value selected – the worst case scenario.

Before arriving at this solution, CEIOPS considered and rejected other possible solutions.

- A second period that CEIOPS defined as being of "very extreme stress" over the last twelve years was the period from February 12, 2008 through May 8, 2008. For this period, the empirical correlation factor was reported as 54%.

- Two other periods considered were those from November 2, 2007 through February 27, 2009 and August 28, 1998 through January 29, 1999. Both of these had much lower correlation factors, to wit, -21% and 32%, respectively.

Similar approaches were used to calculate other correlation factors. The details are presented on pages 357-371 of what was once known as the QIS5 Calibration paper and is now known as the Solvency II Calibration paper.

Testing the Calibration for Market Risk

Finally, CEIOPS tested the resulting SCR for market risk to see if it achieves reasonable results. Specifically, in Paragraphs 3.1337 and 3.1338 of the QIS 5 calibration paper,

[1] Some related methodological issues are discussed in Appendix B of this work.

CEIOPS states that in order "[t]o assess whether a correlation matrix [approach] provides a [market risk] capital [requirement] in line with a stress at the 99.5% VaR, we need to consider the matrix as a whole, and consider whether the diversification benefit it provides is consistent with that we would expect in a 99.5% VaR event."

According to paragraph 3.1276, the goal for the market risk component, is for the absolute error

$$\left|VaR_{mkt} - SCR_{mkt}\right| \qquad (3)$$

to be close to zero. Here, VaR_{mkt} is at the 99.5% confidence level set for one year. These are the typical parameter values selected by CEIOPS.

The details around the calculation of the VaR_{market} are unclear. CEIOPS appears to have created a model ("typical") insurance company and calculated a market risk capital requirement for that company based on actual historical data, then compared the results against a market risk capital requirement determined using the sub-risk requirements and correlation factors. While the details of the model insurance company are not given, the individual market sub-risk SCRs are:

TABLE 3

SCR for Market Risks of "Typical" Company	
Market Risk	Amount of SCR
Interest Rates	29.36
Equity	39.24
Property	8.39
Spread	11.00
Currency	5.22
Concentration	6.80
Total	100.00

In Sections 3.1346 and 3.1347, CEIOPS concludes as follows:

"Under the empirical [i.e., "typical"] model [approach], the 99.5^{th} percentile capital requirement is 82.5, and under the Level 2 proposed matrix, the 99.5^{th} percentile capital requirement is 83.7. The difference is low, with the Level 2 matrix being slightly too [conservative] by approximately 1.2%."

"This indicates that the correlation matrix proposed by CEIOPS provides overall capital figures broadly consistent with a 99.5% VaR".

It is not entirely clear to us how CEIOPS arrived at the 83.7 SCR. Using Equation (2), the SCR's in Table 3, and the correlation factors of Table 2, Tom calculated the 99.5% VaR to be 81.94. This is an issue that we need to investigate further.

OTHER RISK COMPONENTS

The SCRs for other risks were also calibrated, as were the corresponding correlation factors. As will be seen in the next few sections, considerable judgment has been applied in the process.

Life Underwriting Risk Module

The life underwriting risk module includes the following sub-risks:

- mortality risk,
- longevity risk,
- disability risk,
- lapse risk,
- expense risk,
- revision risk[2], and
- catastrophe risk.

According to CEIOPS, "[t]here is no appropriate database for the calibration of the life underwriting risk correlation factors. For the time being, the choice of these factors needs to be based on expert opinion." See Solvency II Calibration paper – page 351 – Paragraph 3.1309.

The correlation factors for this risk module are summarized in the following table:

TABLE 4
CORRELATION MATRIX FOR LIFE UNDERWRITING RISK
(SEE PAGE 353)

	Mortality	Longevity	Disability	Lapse	Expenses	Revision	CAT
Mortality	1						
Longevity	-0.25	1					
Disability	0.25	0	1				
Lapse	0	0.25	0	1			
Expenses	0.25	0.25	0.5	0.5	1		
Revision	0	0.25	0	0	0.5	1	
CAT	0.25	0	0.25	0.25	0.25	0	1

[2] As an example, revision risk might involve the adverse variation of an annuity's amount, as a result of an unanticipated revision of the claims process.

Non-life Underwriting Risk Module

This consists of two sub-modules: (1) the non-life premium and reserve sub-module and (2) the non-life catastrophe module. "The [non-life] premium and reserve risk module also uses correlations between different lines of business (LOB's) to estimate the combined standard deviation of premium and reserve risk."

The correlation factor between (1) catastrophe risk and (2) non-life premium and reserve risk is based on expert opinion. (See the discussion on page 353.) CEIOPS selected this correlation factor to be 0.25.

Health Underwriting Risk Module

"At [the] current [time], there is no appropriate database for the calibration of the health underwriting risk correlation factors. Therefore, for the time being, the choice of these factors needs to be based on expert opinion." CEIOPS – page 355 – Paragraph 3.1327.

CEIOPS assumed that the correlation factors of Table 4 (the entries of the correlation matrix for life underwriting) would also be appropriate for the health underwriting correlation factors for those products that are more similar to life insurance. For those health products not similar to life-insurance, judgment was used to establish correlations across three lines of business: accident, sickness, and workers' compensation.

Comment on Discussion of Health Underwriting Risk Module

The above is based, as noted, on the discussion of Paragraph 3.1327. However, Paragraphs 3.1328 – 3.1334 appear to contradict Paragraph 3.1327. It is not clear which is the intended approach. For ease of exposition, we have limited our discussion to the simpler approach of Paragraph 3.1327.

SUMMARY

In this work, we have attempted to give the reader a feel for the methods that CEIOPS employs to compute its "solvency capital requirements". We started with the overall formula of Equation (1). In Table 1, we presented the overall correlation factors. We then proceeded to discuss the formulas for the individual component risks. These formulas entailed the use of other correlation matrices. Clearly, CEIOPS conducted a huge amount of thoughtful analysis to obtain its results. Not surprisingly, a large proportion of its results rely heavily on "expert opinion", judgment, and subjectivity.

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APPENDIX A

MATHEMATICAL PRELIMINARIES -- BASICS OF VALUE-AT-RISK

At its simplest, Value at Risk (often written VaR) is merely a summary measure of market risk. It provides, in terms of dollars or any other appropriate monetary unit, a number that can be interpreted as an indication of a portfolio's sensitivity to financial market risk.

The theoretical basis for Value at Risk is found in probability theory, while the terminology for VaR comes from the frequentist paradigm of statistics. As with all probabilistic models, the application of VaR entails a number of underlying assumptions. The principal assumption for VaR is about the behavior of the financial markets for the portfolio under consideration. In the majority of cases, the assumption is made that the current market conditions will prevail for the immediate future. Then, simply stated, the VaR of the portfolio is the maximum loss anticipated over a given time period for a specified "confidence interval".

We are now ready to present the basic definition of Value at Risk. We first let

- W denote the portfolio's value at the beginning of the time period of interest
- T denote the length of the time period, and
- V denote the random variable representing the value of the portfolio at the end of the time period.

Then, given a confidence level, α, and the assumption(s) on market conditions, the Value at Risk, VaR, satisfies the equation

$$\alpha = \Pr\left[V \geq W - VaR\right] = \Pr\left[W - V \leq VaR\right]. \qquad (\text{A}-1)$$

To be more specific, if $\alpha = 95\%$ and the length of T is one year, then the last equation states that the probability is 95% that the portfolio will lose no more than VaR dollars over the one-year period.

<u>Example of Use of Value at Risk</u>

Suppose we expect the value of a $100 portfolio one year from today to be closely approximated by a lognormal distribution with parameters $\mu = 4.685$ and $\sigma = .20$. What is the Value at Risk at a 95% confidence level over this one-year period?

Because the random variable V representing the value of the portfolio one year from today is (approximately) lognormal, the $\ln V$ is a normal random variable with mean

4.685 and standard deviation .20. Since the initial value of the portfolio is \$100 million, we can rewrite Equation (A-1) as

$$\Pr\left[V \geq W - VaR\right] = .95,$$

or

$$\Pr\left[\frac{\ln V - 4.685}{.20} \geq \frac{\ln\left(100 - VaR\right) - 4.685}{.20}\right] = .95,$$

or

$$\Pr\left[Z \geq \frac{\ln\left(100 - VaR\right) - 4.685}{.20}\right] = .95, \qquad \text{(A} - 2\text{)}$$

where Z is a standard normal random variable and VaR is in units of millions of dollars. The 5^{th} percentile of the distribution of Z is -1.645, so Equation (A-2) implies that

$$\frac{\ln\left(100 - VaR\right) - 4.685}{.20} = -1.645$$

or

$$\text{VaR} = \$22.055 \text{ million}.$$

In other words, over the one-year period there is a 95% probability that the portfolio will lose no more than \$22.055 million of its initial value of \$100 million.

APPENDIX B

COMPUTATION OF CORRELATION COEFFICIENTS FOR SUB-RISKS

CEIOPS argues that the use of the customary Pearson product-moment measure to estimate the correlation coefficient of the pair of risks (i.e., random variables) X and Y is not appropriate here because

- The dependency between the distributions of the two risks X and Y is typically not linear – e.g., there are tail dependencies. (This means they both have a tendency to go to the tail at the same time.)

- The distributions of X and Y are typically skewed and so they both violate the required normality condition. (I am not sure why this is a problem. The correlation coefficient can always be computed irrespective of the underlying distributions of the random variables.)

For these reasons, CEIOPS suggests choosing the correlation coefficient, ρ_{XY} ,that minimizes the absolute aggregation error

$$\left|VaR(X+Y)^2 - VaR(X)^2 - VaR(Y)^2 - \rho_{XY} \times VaR(X) \times VaR(Y)\right|. \qquad \text{(B-5)}$$

(We note here that CEIOPS has used two notational schemes -- ρ_{XY} and $Corr_{i,j}$ -- to denote correlation coefficients.)

In Section 3.1253, "CEIOPS acknowledges that achieving this conceptual [goal] is likely to present a number of practical [computational] challenges".

- In most cases the VaR can't be estimated directly.

- The parameter, ρ_{XY} , is likely to depend on an insurer's specific insurance risks and so vary across insurers.

- When "more than two risks are aggregated, the minimization of" Equation (B-5) has to be extended to include all of the combinations of risks.